Honors Pre-Calculus – Section 12.1 | **Name** Key
Exploring Limits | **Date** ______ **Per.** ______

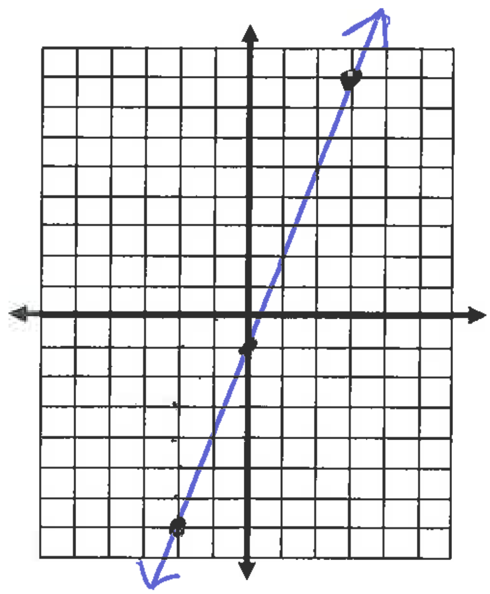
1. Evaluate the function at the indicated values and then plot the points and graph the graph. Then find the indicated limits.

$f(x)=3x-1$

$f(0)=-1$

$f(3)=9-1=8$

$f(-2)=-6-1=-7$



$\lim_{x\to 0}(3x-1)=-1$

$\lim_{x\to 3}(3x-1)=8$

$\lim_{x\to -2}(3x-1)=-7$

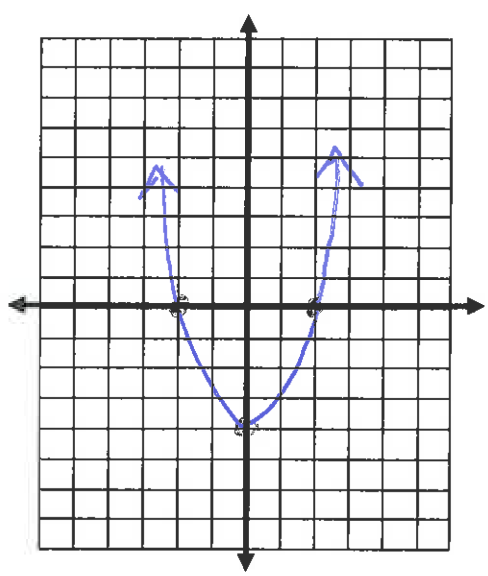
2. Evaluate the function at the indicated values and then plot the points and graph the graph. Then find the indicated limits.

$f(x)=x^2-4$

$f(0)=-4$

$f(2)=0$

$f(-2)=0$



$\lim_{x\to 0}(x^2-4)=-4$

$\lim_{x\to 3}(x^2-4)=9-4=5$

$\lim_{x\to -2}(x^2-4)=0$

<u>Theorem: The Existence of a Limit</u>

$\lim_{x\to c} f(x)=L$ iff $\lim_{x\to c^+} f(x)=\lim_{x\to c^-} f(x)=L$ (i.e. the right and left hand limits are the same). On the functions domain.

3. Graph the given function and then find the indicated limits.

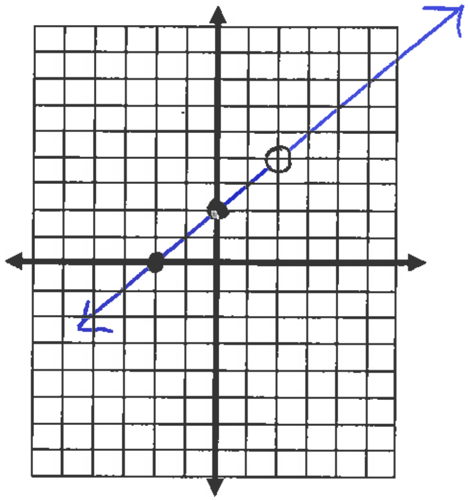
$$y=\left(\frac{x^2-4}{x-2}\right)$$

$$=\frac{(x-2)(x+2)}{(\cancel{x-2})}$$

$y = x+2$

$x \neq 2$

$(2,4)$ is a Point Discontinuity



$\lim\limits_{x\to 0}\left(\frac{x^2-4}{x-2}\right) = \lim\limits_{x\to 0} x+2 = 2$

$\lim\limits_{x\to 2}\left(\frac{x^2-4}{x-2}\right) = \lim\limits_{x\to 2} x+2 = 4$

$\lim\limits_{x\to -2}\left(\frac{x^2-4}{x-2}\right) = \lim\limits_{x\to -2} x+2 = 0$

4. Graph the given function and then find the indicated limits.

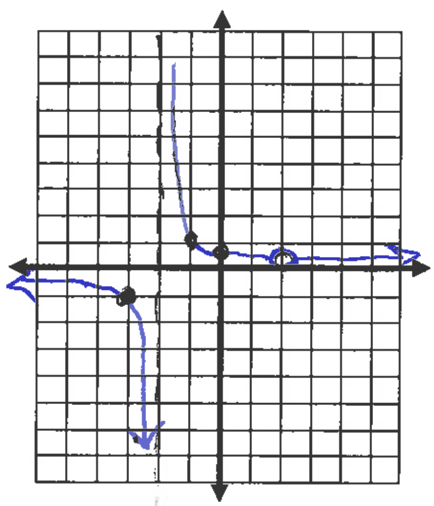
$$y=\left(\frac{x-2}{x^2-4}\right)$$

$$y=\frac{\cancel{x-2}}{(\cancel{x-2})(x+2)}$$

$y = \frac{1}{x+2}$

$x \neq 2$ pd $(2, \frac{1}{4})$

$x \neq -2$ asymptote



$\lim\limits_{x\to 0}\left(\frac{x-2}{x^2-4}\right) = \lim\limits_{x\to 0} \frac{1}{x+2} = \frac{1}{2}$

$\lim\limits_{x\to 2}\left(\frac{x-2}{x^2-4}\right) = \lim\limits_{x\to 2} \frac{1}{x+2} = \frac{1}{4}$

$\lim\limits_{x\to -2}\left(\frac{x-2}{x^2-4}\right) = \lim\limits_{x\to -2} \frac{1}{x+2} = \frac{1}{0}$ undefined limit DNE

$\lim\limits_{x\to \infty}\left(\frac{x-2}{x^2-4}\right) = \lim\limits_{x\to \infty} \frac{1}{x+2} = 0$

5. Find the indicated limits by direct substitution verify graphically.

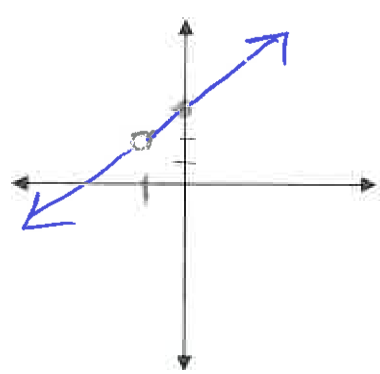
$\lim\limits_{x\to 0}\left(\frac{x^2+4x+3}{x+1}\right) = 3$

$\lim\limits_{x\to 0} \frac{(x+3)(\cancel{x+1})}{\cancel{x+1}}$

$\lim\limits_{x\to 0} x+3 = 3$

$\lim\limits_{x\to -1}\left(\frac{x^2+4x+3}{x+1}\right) =$

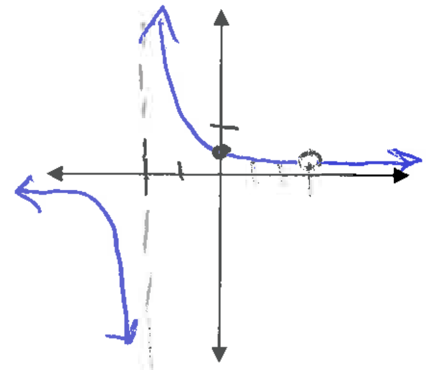
$\lim\limits_{x\to -1} x+3 = 2$



6. Find the indicated limits by direct substitution verify graphically.

$y = \frac{x-3}{(x-3)(x+2)} = \frac{1}{x+2} \quad x \neq 3$

$\lim_{x\to 0}\left(\frac{x-3}{x^2-x-6}\right) = \frac{0-3}{0-0-6} = \frac{-3}{-6} = \frac{1}{2}$



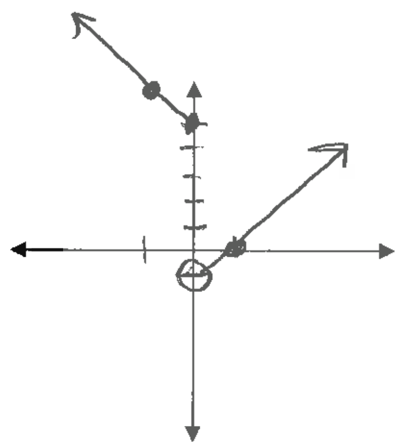
$\lim_{x\to 3}\left(\frac{x-3}{x^2-x-6}\right) = \frac{0}{0}$ = indeterminate

$= \lim_{x\to 3} \frac{1}{x+2} = \frac{1}{5} \quad \left(3, \frac{1}{5}\right)$ ↗ P.D.

$\lim_{x\to \infty}\left(\frac{x-3}{x^2-x-6}\right) = 0$

7. Sketch the graph and analyze the limits at the indicated points

$$g(x) = \begin{cases} -x+5 & x \le 0 \\ x-1 & x > 0 \end{cases}$$



a. $\lim_{x\to -2} g(x) = -(-2)+5 = 7$

b. $\lim_{x\to 3} g(x) = 3-1 = 2$

c. $\lim_{x\to 0} g(x) =$ DNE $\lim_{x\to 0^+} \neq \lim_{x\to 0^-}$

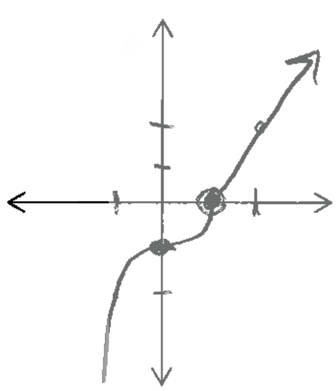
8) Graph $f(x) = \begin{cases} x^3-1, & x<1 \\ 2x-2, & x \ge 1 \end{cases}$

Find: $\lim_{x\to 1} f(x) = 0$

$\lim_{x\to 1^+} f(x) = 0$

$\lim_{x\to 1^-} f(x) = 0$

$\therefore \lim_{x\to 1} f(x) = 0$



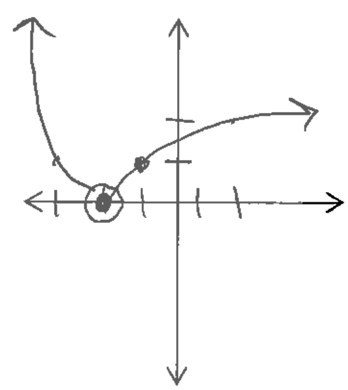
9) Graph $h(x) = \begin{cases} \sqrt{x+2}, & x \ge -2 \\ (x+2)^2, & x < -2 \end{cases}$

Find: $\lim_{x\to -2} h(x) = 0$

$\lim_{x\to -2^+} h(x) = 0$

$\lim_{x\to -2^-} h(x) = 0$

$\therefore \lim_{x\to -2} h(x) = 0$



Definition of Continuity: ("The Continuity Test)

Continuity at a Point: A function f is **continuous at c** if the following three conditions are met.

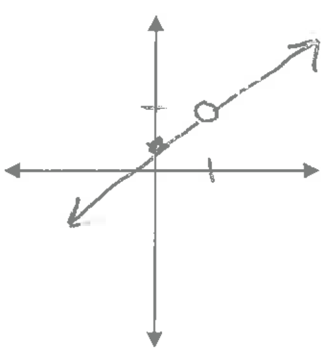
1. $f(c)$ is defined. $\longrightarrow$ there is a point @ $x=c$
2. $\lim_{x\to c} f(x)$ exists. $\longrightarrow$ there is a limit @ $x=c$
3. $\lim_{x\to c} f(x) = f(c)$. $\longrightarrow$ the point @ $x=c$ = lim @ $x=c$

10. Sketch and determine which function is continuous at $x=1$. Why?

a. $g(x)=\dfrac{x^2-1}{x-1} = x+1 \quad x \neq 1$

Test

① f(1) DNE PD



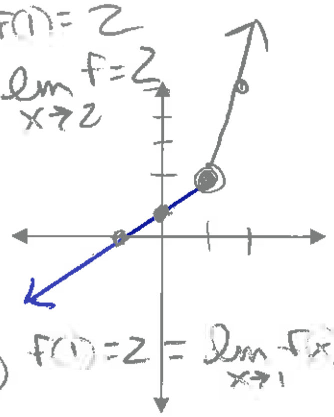
Not Continuous @ $x=1$

b. $h(x)=\begin{cases} x+1, & x\le 1 \\ 3x-1 & x>1 \end{cases}$

test

① f(1) = 2

② $\lim_{x\to 2} f = 2$



③ $f(1)=2 = \lim_{x\to 1} f(x)$ ✓

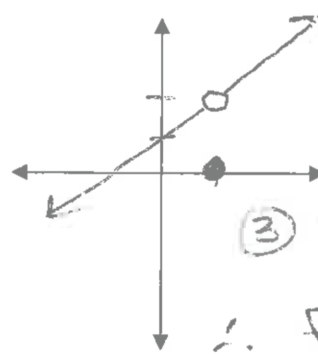
Continuous @ $x=1$

c. $j(x)=\begin{cases} \dfrac{x^2-1}{x-1}, & x\neq 1 \\ 0 & x=1 \end{cases} = x+1$

Test

① f(1) = 0

② $\lim_{x\to 1} f = 2$



③ $f(1) \neq \lim_{x\to 1} f$

∴ f is Not continuous @ $x=1$

11. Sketch the following then apply the **continuity test** at the indicated point:

a. Apply the **continuity test** at x = 0.

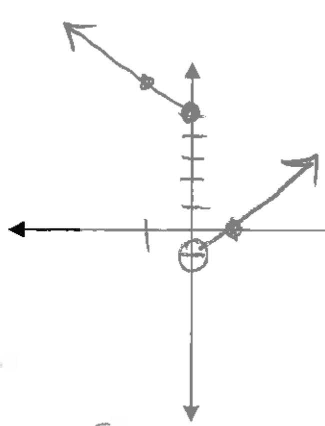
$g(x)=\begin{cases} -x+5 & x\le 0 \\ x-1 & x>0 \end{cases}$

Test

① g(0) = 5

② $\lim_{x\to 5} g(x)$ = DNE

$\lim_{x\to 5^-} g \neq \lim_{x\to 5^+} g$



∴ g is not cont @ $x=0$

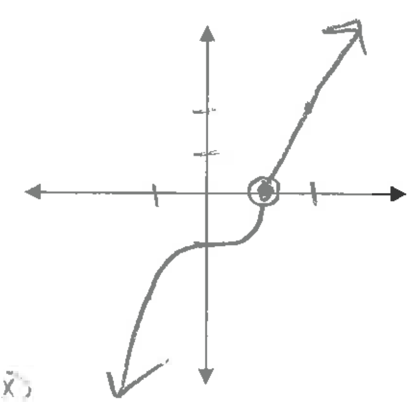
b. Apply the **continuity test** at x = 1.

$f(x)=\begin{cases} x^3-1, & x<1 \\ 2x-2, & x\ge 1 \end{cases}$

Test

① f(1) = 0

② $\lim_{x\to 1} f(x) = 0$



③ $f(1) = \lim_{x\to 1} f(x)$

∴ f(x) is Cont @ $x=1$

12. Evaluate the following limits given the following: $\lim_{x \to c} f(x) = 27$ $\lim_{x \to c} g(x) = 12$

a. $\lim_{x \to c} \sqrt[3]{f(x)} = \sqrt[3]{\lim_{x \to c} f(x)}$

$= \sqrt[3]{27} \boxed{= 3}$

b. $\lim_{x \to c} \frac{f(x)}{18} = \frac{\lim_{x \to c} f(x)}{18} = \frac{27}{18} \boxed{= \frac{3}{2}}$

c. $\lim_{x \to c} \left[f(x) \cdot g(x) \right]$

$= \lim_{x \to 2} f(x) \cdot \lim_{x \to 2} g(x)$

$= 27 \cdot 12 \boxed{= 324}$

d. $\lim_{x \to c} \left[f(x) - 2 \cdot g(x) \right]$

$= \lim_{x \to c} f(x) - 2 \lim_{x \to c} g(x)$

$= 27 - 24 \boxed{= 3}$

13. Evaluate the following limits:

a. $\lim_{x \to 1} \left(\frac{2x^2 - 5x - 3}{x - 3} \right) =$

$= \frac{2 - 5 - 3}{1 - 3} = \frac{-6}{-2}$

$\boxed{= 3}$

b. $\lim_{x \to 6} \left(\frac{\frac{1}{5} + \frac{1}{x-5}}{x} \right) =$

$= \frac{\frac{1}{5} + \frac{1}{6-5}}{6} = \frac{\frac{1}{5} + 1}{6}$

$\boxed{= \frac{\frac{6}{5}}{6} = \frac{36}{5}}$

c. $\lim_{x \to 7} \left(\frac{\sqrt{x+9} - 3}{x} \right) =$

$= \frac{\sqrt{16} - 3}{7} = \frac{4-3}{7}$

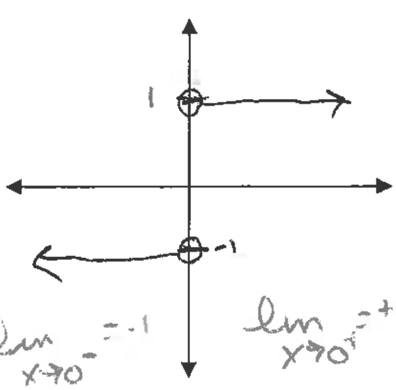
$\boxed{= \frac{1}{7}}$

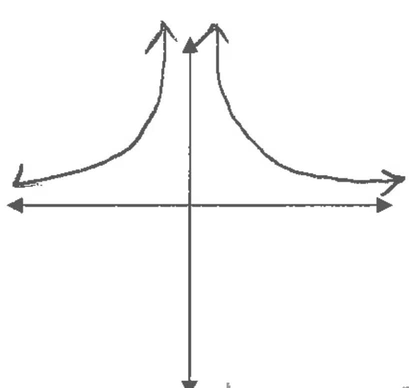
13. Find the limits by analyzing their graph on a calculator.

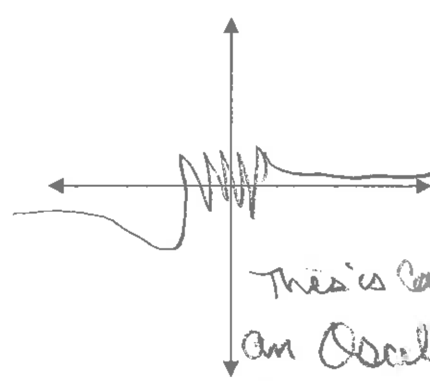
a. $\lim_{x \to 0} \left(\frac{|x|}{x} \right)$ $\boxed{\text{JUMP DISC}}$

b. $\lim_{x \to 0} \left(\frac{1}{x^2} \right)$

c. $\lim_{x \to 0} \left[\sin\left(\frac{1}{x} \right) \right]$







$\lim_{x \to 0^-} = -1$ $\lim_{x \to 0^+} = +1$

$\lim_{x \to 0} \frac{|x|}{x} = DNE$

$\lim_{x \to 0^-} \neq \lim_{x \to 0^+}$

$\lim_{x \to 0} \frac{1}{x^2} = DNE$

f(x) has an asymptote @ x=0

This is called an Oscillating Discontinuit

$\lim_{x \to 0} \sin(\frac{1}{x}) = DNE$

oscillating

Honors Pre-Calculus – Section 12.2 | **Name** ______________________
Evaluating Limits | **Date** ______________ **Per.** ________

1. Evaluate the following limits:

a. $\lim_{x\to 3}\left(\dfrac{x^2+11x+30}{x+5}\right) = 9$

$= \dfrac{9+33+30}{8} = \dfrac{72}{8} = 9$

b. $\lim_{x\to 1}\left(\dfrac{\frac{1}{3}-\frac{1}{x+3}}{x}\right) = \dfrac{1}{12}$

$= \dfrac{\frac{1}{3}-\frac{1}{4}}{1} = \dfrac{4-3}{12}$

$\boxed{=\dfrac{1}{12}}$

c. $\lim_{x\to 7}\left(\dfrac{\sqrt{x+9}-3}{x}\right) = \dfrac{1}{7}$

$= \dfrac{\sqrt{16}-3}{7} = \dfrac{4-3}{7}$

$\boxed{=\dfrac{1}{7}}$

2. Evaluate the following:

a. $\lim_{x\to -5}\left(\dfrac{x^2+11x+30}{x+5}\right) = \dfrac{25-55+30}{0}\ \boxed{=\dfrac{0}{0}\ \text{IND}}$

$= \lim_{x\to -5}\dfrac{\cancel{(x+5)}(x+6)}{\cancel{x+5}}$

$\boxed{= \lim_{x\to -5} x+6 = +1}$

b. $\lim_{x\to 3}\left(\dfrac{x^2-x-6}{x-3}\right) = \dfrac{9-3-6}{0} = \dfrac{0}{0}$ IND

$= \lim_{x\to 3}\dfrac{\cancel{(x-3)}(x+2)}{\cancel{x-3}}$

$\boxed{= \lim_{x\to 3} x+2 = 5}$

c. $\lim_{x\to 0}\left(\dfrac{\sqrt{x+9}-3}{x}\right) = \dfrac{3-3}{0}\ \boxed{=\dfrac{0}{0}\ \text{IND}}$

$\lim_{x\to 0}\dfrac{\sqrt{x+9}-3}{x}\cdot\dfrac{(\sqrt{x+9}+3)}{\sqrt{x+9}+3} = \dfrac{x+9-9}{x(\sqrt{x+9}+3)}$

$\lim_{x\to 0}\dfrac{\cancel{x}}{\cancel{x}(\sqrt{x+9}+3)} = \lim_{x\to 0}\dfrac{1}{\sqrt{x+9}+3}$

$\boxed{= \dfrac{1}{3+3} = \dfrac{1}{6}}$

d. $\lim_{x\to 0}\left(\dfrac{\sqrt{5-x}-\sqrt{5}}{x}\right) = \dfrac{\sqrt{5}-\sqrt{5}}{0}\ \boxed{=\dfrac{0}{0}\ \text{IND}}$

$\lim_{x\to 0}\dfrac{(\sqrt{5-x}-\sqrt{5})}{x}\cdot\dfrac{(\sqrt{5-x}+\sqrt{5})}{\sqrt{5-x}+\sqrt{5}}$

$= \lim_{x\to 0} = \dfrac{5-x-5}{x(\sqrt{5-x}+\sqrt{5})} = \dfrac{-x}{x(\sqrt{5-x}+\sqrt{5})}$

$= \lim_{x\to 0}\dfrac{-1}{\sqrt{5-x}+\sqrt{5}}$

$= \dfrac{-1}{\sqrt{5}+\sqrt{5}} = \boxed{\dfrac{-1}{2\sqrt{5}}}$

e. $\lim_{x\to 0^{\pm}} f(x)$ where $f(x)=\begin{cases}2x+1 & \text{if } x\ge 0\\ 2x-1 & \text{if } x<0\end{cases}$

$\lim_{x\to 0^+} f(x) = 2(0)+1 = 1$

$\lim_{x\to 0^-} f(x) = 2(0)-1 = -1$

f. $\lim_{x\to 0}\left(\frac{\frac{1}{3}-\frac{1}{x+3}}{x}\right) = \frac{\frac{1}{3}-\frac{1}{3}}{0} = \boxed{\frac{0}{0}\ \text{IND}}$

$= \lim_{x\to 0} \frac{\frac{1}{3}-\frac{1}{x+3}}{x} = \lim_{x\to 0} \frac{x+3-3}{(x+3)(3)} \cdot \frac{1}{x}$

$= \lim_{x\to 0} \frac{\cancel{x}}{(x+3)(3)} \cdot \frac{1}{\cancel{x}}$

$= \boxed{\frac{1}{3\cdot 3} = \frac{1}{9}}$

g. $\lim_{x\to 0}\left(\frac{\frac{1}{4-x}-\frac{1}{4}}{x}\right) = \frac{\frac{1}{4}-\frac{1}{4}}{0} = \boxed{\frac{0}{0}\ \text{IND}}$

$= \lim_{x\to 0} \frac{4-(4-x)}{(4-x)(4)} \cdot \frac{1}{x} = \lim_{x\to 0} \frac{4-4+x}{(4-x)(4)} \cdot \frac{1}{x}$

$= \lim_{x\to 0} \frac{\cancel{x}}{(4-x)(4)} \cdot \frac{1}{\cancel{x}} = \frac{1}{4(4)}$

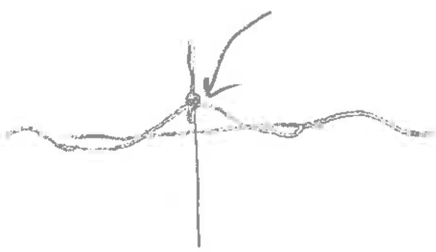
$\boxed{= \frac{1}{16}}$

Theorem: The Existence of a Limit

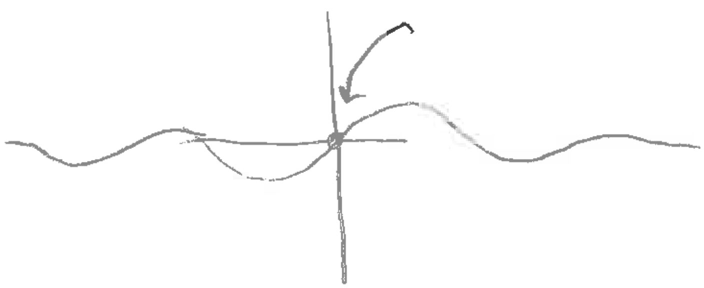
$\lim_{x\to c} f(x) = L$ iff $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = L$ (i.e. the right and left hand limits are the same)

3. Evaluate the following (by graphing)

a. $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1$



b. $\lim_{x\to 0}\left(\frac{1-\cos x}{x}\right) = 0$



4. Find $\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ for the following:

a. $f(x) = 3x+1$

$\lim_{h\to 0} \frac{3(x+h)+1-(3x+1)}{h}$

$\lim_{h\to 0} \frac{3x+3h+1-3x-1}{h}$

$\boxed{\lim_{h\to 0} \frac{3h}{h} = 3}$

b. $f(x) = x^2+2$

$\lim_{h\to 0} \frac{(x+h)^2+2-(x^2+2)}{h}$

$\lim_{h\to 0} \frac{x^2+2xh+h^2+2-x^2-2}{h}$

$\boxed{\lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x+0 = 2x}$